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ON τ-CENTRALIZERS OF SEMIPRIME RINGS

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Abstract: Let R be a semiprime 2-torsion free ring, and let τ be an endomorphism of R. Under some conditions we prove that a left Jordan τ-centralizer of R is a left τ-centralizer of R. Under the same conditions we also prove that a Jordan τ-centralizer of R is a τ-centralizer of R. We thus generalize Zalar's results to the case of τ-centralizers of R.

Keywords: prime ring, semiprime ring, left centralizer, left Jordan centralizer, left τ-centralizer, left Jordan τ-centralizer, generalized (σ,τ)-derivation, generalized Jordan (σ,τ)-derivation

1. Introduction

Throughout this article, R represents an associative ring with center Z. Recall that R is prime if $xRy = 0$ implies either $x = 0$ or $y = 0$, and R is semiprime if $xRx = 0$ implies $x = 0$. An additive mapping $\alpha : R \to R$ is called a *derivation* if $\alpha(xy) = \alpha(x)y + x\alpha(y)$ holds for all $x, y \in R$. An additive mapping $\alpha : R \to R$ is called a *Jordan derivation* if $\alpha(x^2) = \alpha(x)x + x\alpha(x)$ holds for all $x \in R$. It is clear that each derivation of R is a Jordan derivation. The converse is false in general. Herstein's result [1] states that each Jordan derivation of a prime 2-torsion free ring is a derivation. M. Brešar [2] extended this result to the case of Jordan derivations of semiprime 2-torsion free rings. Given some endomorphisms σ and τ of R, an additive mapping $\alpha : R \to R$ is called a *(σ,τ)-derivation* if $\alpha(xy) = \alpha(x)\tau(y) + \sigma(x)\alpha(y)$ holds for all $x, y \in R$. Recall that a *Jordan (σ,τ)-derivation*, as defined in [3], is an additive mapping $\alpha : R \to R$ satisfying $\alpha(x^2) = \alpha(x)\tau(x) + \sigma(x)\alpha(x)$ for all $x \in R$. In [3], under some conditions it was shown that each Jordan (σ,τ)-derivation of a prime 2-torsion free ring R is a (σ,τ)-derivation.

Recently, M. Brešar [4] introduced the following definition: An additive mapping $d : R \to R$ is called a *generalized derivation* if there exists a derivation α of R such that $d(xy) = d(x)y + x\alpha(y)$ for all $x, y \in R$. Hence, the notion of generalized derivation includes the notions of derivation and left multiplier (i.e. an additive mapping satisfying $f(xy) = f(x)y$ for all $x, y \in R$). The main examples are the derivations and the generalized inner derivations (i.e. the mappings of the type $x \mapsto ax + xb$ for some $a, b \in R$). Given an arbitrary mapping $d : R \to R$ and an additive mapping $\alpha : R \to R$ of a semiprime (or prime) ring R such that $d(xy) = d(x)y + x\alpha(y)$ for all $x, y \in R$, we note that d is uniquely defined by α, which should be a derivation by [2, Remark 1]. Following [5], an additive mapping $d : R \to R$ is called a *generalized Jordan derivation* if there is a derivation α of R such that $d(x^2) = d(x)x + x\alpha(x)$ for all $x \in R$. It is clear that each generalized derivation of R is a generalized Jordan derivation. In [6], it was proved that each generalized Jordan derivation of a prime 2-torsion free ring is a generalized derivation. The schemes of the proofs of these results may be found in [7] (also see [2, 3, 8] and [9], where some generalizations may be found). Given some endomorphisms σ and τ of R, an additive mapping $d : R \to R$ is called a *generalized (σ,τ)-derivation* if there is a (σ,τ)-derivation α of R such that $d(xy) = d(x)\tau(y) + \sigma(x)\alpha(y)$ for all $x, y \in R$.

An additive mapping $d : R \to R$ is called a *generalized Jordan (σ,τ)-derivation* if there is a (σ,τ)-derivation α of R such that $d(x^2) = d(x)\tau(x) + \sigma(x)\alpha(x)$ for all $x \in R$. It is clear that each (σ,τ)-derivation is a generalized Jordan (σ,τ)-derivation.

B. Zalar [10] introduced the following notion. Let R be a semiprime ring. A *left* (*right*) *centralizer* of R is an additive mapping $T : R \to R$ satisfying $T(xy) = T(x)y$ ($T(xy) = xT(y)$) for all $x, y \in R$. If T is a left and a right centralizer then T is a *centralizer*. An additive mapping $T : R \to R$ is called

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a *Jordan centralizer* if T satisfies $T(xy + yx) = T(x)y + yT(x) = T(y)x + xT(y)$ for all $x, y \in R$. A *left* (*right*) *Jordan centralizer* of R is an additive mapping $T : R \to R$ such that $T(x^2) = T(x)x$ ($T(x^2) = xT(x)$) for all $x \in R$. In [10], it was shown that a Jordan centralizer of a semiprime ring is a left centralizer, and each Jordan centralizer is a centralizer.

Introduce the following definitions: Let R be a semiprime 2-torsion free ring, and let τ be an endomorphism of R. A *Jordan τ-centralizer* of R is an additive mapping $f : R \to R$ satisfying

$$f(xy + yx) = f(x)\tau(y) + \tau(y)f(x) = f(y)\tau(x) + \tau(x)f(y)$$

for all $x, y \in R$. An additive mapping $f : R \to R$ is called a *left* (*right*) *τ-centralizer* of R if

$$f(xy) = f(x)\tau(y) \quad (f(xy) = \tau(x)f(y))$$

for all $x, y \in R$. If f is a left and a right τ-centralizer then it is natural to call f a *τ-centralizer*. An additive mapping $f : R \to R$ is called a *left* (*right*) *Jordan τ-centralizer* of R if

$$f(x^2) = f(x)\tau(x) \quad (f(x^2) = \tau(x)f(x))$$

for all $x \in R$. It is clear that a left τ-centralizer of R is a left Jordan τ-centralizer and, analogously, a τ-centralizer of R is a Jordan τ-centralizer of R. The converse is false in general. The main aim of the present article is a generalization of Zalar's results to the case of τ-centralizers of R.

2. The First Result

In what follows, R is a semiprime 2-torsion ring and τ is a surjective endomorphism of R. In this section, under some conditions we demonstrate that each left Jordan τ-centralizer of R is a left τ-centralizer.

We denote by $[x, y]$ the commutator $xy - yx$. Following [10], we introduce the notation $D(x, y) = f(xy) - f(x)\tau(y)$ for all $x, y \in R$. Note that D is a biadditive mapping, since D is linear in x and y. If $D(x, y) = 0$ for all $x, y \in R$ then f is a left τ-centralizer of R.

We start with the following results which are essentially used in the proof of our first claim.

Lemma 1 [10, Lemma 1.1]. *Let R be a semiprime ring. If $a, b \in R$ are such that $axb = 0$ for all $x \in R$ then $ab = ba = 0$.*

Lemma 2 [10, Lemma 1.2]. *Let R be a semiprime ring, and let $A, B : R \times R \to R$ be some biadditive mappings. If $A(x, y)wB(x, y) = 0$ for all $w, x, y \in R$ then $A(x, y)wB(u, v) = 0$ for all $u, v, w, x, y \in R$.*

Lemma 3. *Let $f : R \to R$ be a left Jordan τ-centralizer of R. Then*
(i) *$f(xy + yx) = f(x)\tau(y) + f(y)\tau(x)$ for all $x, y \in R$;*
(ii) *$f(xyx) = f(x)\tau(y)\tau(x)$ for all $x, y \in R$;*
(iii) *$f(xyz + zyx) = f(x)\tau(y)\tau(z) + f(z)\tau(y)\tau(x)$ for all $x, y, z \in R$;*
(iv) *$D(x, y) = -D(y, x)$ for all $x, y \in R$;*
(v) *$D(x, y)\tau(z)[\tau(u), \tau(v)] = 0$ for all $u, v, x, y, z \in R$.*

Proof. (i) Since f is a left Jordan τ-centralizer of R, the equality

$$f(x^2) = f(x)\tau(x) \tag{1}$$

holds for all $x \in R$. By linearity in x, from (1) we derive

$$f((x + y)^2) = f(x + y)\tau(x + y) = f(x)\tau(x) + f(y)\tau(y) + f(x)\tau(y) + f(y)\tau(x)$$

for all $x, y \in R$. On the other hand, from (1) we have

$$f((x + y)^2) = f(x^2 + y^2 + xy + yx) = f(x)\tau(x) + f(y)\tau(y) + f(xy + yx)$$

for all $x, y \in R$. Comparing these two obtained relations, we see that (i) holds.

(ii) Replacing y with $xy + yx$ in (i) and using (1), we see that

$$\begin{aligned} f(x(xy+yx)+(xy+yx)x) &= f(x)\tau(xy+yx) + f(xy+yx)\tau(x) \\ &= f(x)\tau(x)\tau(y) + 2f(x)\tau(y)\tau(x) + f(y)\tau(x)\tau(x) \end{aligned}$$

for all $x, y \in R$ and

$$\begin{aligned} f(x(xy+yx)+(xy+yx)x) &= f(x^2y + yx^2 + 2xyx) \\ &= f(x)\tau(x)\tau(y) + f(y)\tau(x)\tau(x) + 2f(xyx) \end{aligned}$$

for all $x, y \in R$. These two relations lead to (ii).

(iii) Replacing x with $x + z$ in (ii), we obtain (iii).

(iv) It is obvious from (i): we have to rewrite (i), using $D(x, y) = f(xy) - f(x)\tau(y)$.

(v) We compute $j = f(xyzyx + yxzxy)$ in two different ways. By (ii),

$$j = f(x)\tau(y)\tau(z)\tau(y)\tau(x) + f(y)\tau(x)\tau(z)\tau(x)\tau(y) \tag{2}$$

for all $x, y, z \in R$. By (iii),

$$j = f(xy)\tau(z)\tau(y)\tau(x) + f(yx)\tau(z)\tau(x)\tau(y) \tag{3}$$

for all $x, y, z \in R$. Comparing (2) and (3), we arrive at

$$D(x, y)\tau(z)\tau(y)\tau(x) + D(y, x)\tau(z)\tau(x)\tau(y) = 0$$

for all $x, y, z \in R$. Therefore, by (iv), $D(x, y)\tau(z)[\tau(y), \tau(x)] = 0$ for all $x, y, z \in R$. Using Lemmas 2 and 1, we infer $D(x, y)\tau(z)[\tau(u), \tau(v)] = 0$ for all $u, v, x, y, z \in R$.

Lemma 4. *The element $D(x, y)$ belongs to Z for all $x, y \in R$.*

Proof. By Lemma 3 (v),

$$\begin{aligned} &[D(x, y), \tau(r)]\tau(z)[D(x, y), \tau(r)] \\ &= D(x, y)\tau(r)\tau(z)[D(x, y), \tau(r)] - \tau(r)D(x, y)\tau(z)[D(x, y), \tau(r)] = 0 \end{aligned}$$

for all $r, x, y, z \in R$. Since R is semiprime and τ is surjective, we have $[D(x, y), \tau(r)] = 0$ for all $r, x, y \in R$. Hence, $D(x, y) \in Z$ for all $x, y \in R$.

Theorem 1. *If $\tau(Z) = Z$ then each left Jordan τ-centralizer of R is a left τ-centralizer.*

Proof. Our aim is to prove that $D(x, y) = 0$ for all $x, y \in R$. Replacing x with xa, for some $a \in Z$, in Lemma 3 (i) and using Lemma 3 (i), we obtain

$$f(xay + yxa) = f(xa)\tau(y) + f(y)\tau(xa) = f(xa)\tau(y) + f(y)\tau(x)\tau(a)$$

for all $x, y \in R$ and $a \in Z$. Since $f(xay+yxa) = f(xay+yax)$ for all $x, y \in R$ and $a \in Z$, by Lemma 3 (iii) we have

$$f(xay + yax) = f(x)\tau(ay) + f(y)\tau(ax) = f(x)\tau(a)\tau(y) + f(y)\tau(a)\tau(x)$$

for all $x, y \in R$ and $a \in Z$. From the last two relations we have $(f(xa) - f(x)\tau(a))\tau(y) = 0$ for all $x, y \in R$ and $a \in Z$. Since R is semiprime and τ is surjective, we infer that

$$f(xa) - f(x)\tau(a) = D(x, a) = 0 \tag{4}$$

for all $x \in R$ and $a \in Z$. It is clear that $D(x, a) = -D(a, x)$ by Lemma 3 (iv). Hence,

$$D(a, x) = f(ax) - f(a)\tau(x) = 0 \tag{5}$$

for all $x \in R$ and $a \in Z$.

We assert now that $D(x,y)c = 0$ for all $x, y \in R$ and $c \in Z$. Since $\tau(Z) = Z$ by the hypothesis of the theorem, we may take $a \in Z$ such that $c = \tau(a)$. Expanding $D(x,y)$, we obtain

$$D(x,y)c = f(xy)\tau(a) - f(x)\tau(y)\tau(a) \tag{6}$$

for all $x \in R$ and $a \in Z$. On the other hand, replacing x with xy in (4), we arrive at $D(xy,a) = f(xya) - f(xy)\tau(a) = 0$ for all $x, y \in R$ and $a \in Z$. Hence,

$$f(xy)\tau(a) = f(xya) \tag{7}$$

for all $x, y \in R$ and $a \in Z$.

By (4),

$$f(x)\tau(a) = f(xa) \tag{8}$$

for all $x, y \in R$ and $a \in Z$. Using (4), (5), (7), (8), and (6) consecutively, we deduce

$$\begin{aligned} D(x,y)c &= f(xy)\tau(a) - f(x)\tau(y)\tau(a) = f(xya) - f(x)\tau(a)\tau(y) \\ &= f(xya) - f(xa)\tau(y) = f(xya) - f(ax)\tau(y) = f(xya) - f(a)\tau(x)\tau(y) \\ &= f(xya) - f(a)\tau(xy) = f(xya) - f(axy) = f(xya - axy) = 0 \end{aligned}$$

for all $x, y \in R$ and $a \in Z$. Hence, by hypothesis $\tau(Z) = Z$, $D(x,y)\tau(a) = D(x,y)c = 0$ for all $x, y \in R$ and $c \in Z$. In particular, we have $D(x,y)D(x,y) = (D(x,y))^2 = 0$ for all $x, y \in R$ by Lemma 4. Since R is semiprime, we obtain the required result $D(x,y) = 0$ for all $x, y \in R$.

Corollary 1. *Let R be a semisimple 2-torsion free ring. If $\tau(Z) = Z$ then each left Jordan τ-centralizer of R is a left τ-centralizer.*

The proof is obvious from Theorem 1 and the well-known fact that every semisimple ring is semiprime.

It is clear that each left τ-centralizer of R is a left Jordan τ-centralizer. However, the following example shows that the converse is false in general.

EXAMPLE 1. Let R be a ring with $\operatorname{char} R \neq 2$ and $a^2 = 0$ for all $a \in R$. Suppose that R contains a nonzero element x such that $axa = 0$ for all $a \in R$ and $axb \neq 0$ for some nonzero $a, b \in R$; moreover, $ax \neq 0$ for some nonzero $a \in R$. Consider the matrix ring $A = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : a, b \in R \right\}$. Let τ be an endomorphism of A defined by $\tau\left(\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$. Then the mapping $f : A \to A$, defined by the rule

$$f\left(\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} ax & ax - a \\ 0 & 0 \end{bmatrix},$$

gives rise to a left Jordan τ-centralizer which is not a left τ-centralizer on A.

3. The Second Result

Recall that R is a semiprime 2-torsion free ring, and σ and τ are some epimorphisms of R. Under some conditions we prove that each Jordan τ-centralizer of R is a τ-centralizer.

Lemma 5. *Let D be a generalized (σ, τ)-derivation of R, and let a be a fixed element in R.*
(i) *If $D(x)D(y) = 0$ for all $x, y \in R$ then $D = 0$.*
(ii) *If $a\tau(x) - \tau(x)a \in Z$ for all $x \in R$ then $a \in Z$.*

PROOF. (i) Let $D(x)D(y) = 0$ for all $x, y \in R$. Replacing y with yz, we obtain

$$0 = D(x)D(yz) = D(x)D(y)\tau(z) + D(x)\sigma(y)\alpha(z) = D(x)\sigma(y)\alpha(z)$$

for all $x, y \in R$. Since σ is surjective and R is semiprime, by Lemma 1

$$\alpha(z)D(x) = 0 \tag{9}$$

for all $x, z \in R$. Replacing x with xz in (9), we have

$$0 = \alpha(z)D(xz) = \alpha(z)D(x)\tau(z) + \alpha(z)\sigma(x)\alpha(z)$$

for all $x, z \in R$. Since R is semiprime and σ is surjective, we have $\alpha = 0$. Then

$$0 = D(zx)D(z) = D(z)\tau(x)D(z) + \sigma(z)\alpha(x)D(z) = D(z)\tau(x)D(z)$$

for all $x, z \in R$. Since τ is surjective (and R is semiprime), the last equality implies $D = 0$, as required.

(ii) Put $D(x) = a\tau(x) - \tau(x)a$. It is easy to see that D is a (τ, τ)-derivation of R, i.e.,

$$D(xy) = a\tau(xy) - \tau(xy)a = a\tau(x)\tau(y) - \tau(x)\tau(y)a$$
$$= (a\tau(x) - \tau(x)a)\tau(y) + \tau(x)(a\tau(y) - \tau(y)a) = D(x)\tau(y) + \tau(x)D(y)$$

for all $x, y \in R$. It is clear that $D(y) \in Z$ for all $y \in R$ by (ii). Thus, we have $D(y)x = xD(y)$ and, analogously, $D(yz)x = xD(yz)$ for all $x, y, z \in R$. Hence,

$$D(y)\tau(z)x + \tau(y)D(z)x = xD(y)\tau(z) + x\tau(y)D(z),$$
$$D(y)(\tau(z)x - x\tau(z)) = D(z)(x\tau(y) - \tau(y)x)$$

for all $x, y, z \in R$. Take y instead of z and a instead of x. Then

$$0 = 2D(y)(\tau(y)a - a\tau(y)) = 2D(y)D(y)$$

for all $y \in R$. Since R is a semiprime 2-torsion free ring, we have $D(y)^2 = 0$ for all $y \in R$. Thus, since R is semiprime, (ii) implies $D(y) = 0$ for all $y \in R$. Since τ is surjective and $0 = D(y) = \tau(y)a - a\tau(y)$ for all $y \in R$, we infer that $a \in Z$.

Lemma 6. *Let a be a fixed element in R, and $f(x) = a\tau(x) + \tau(x)a$ (for all $x \in R$). If f is a Jordan τ-centralizer of R then $a \in Z$.*

PROOF. Since f is a Jordan τ-centralizer of R, we have $f(xy + yx) = f(x)\tau(y) + \tau(y)f(x)$ for all $x \in R$. Hence,

$$a\tau(xy + yx) + \tau(xy + yx)a = (a\tau(x) + \tau(x)a)\tau(y) + \tau(y)(a\tau(x) + \tau(x)a),$$
$$a\tau(y)\tau(x) + \tau(x)\tau(y)a = \tau(x)a\tau(y) + \tau(y)a\tau(x)$$

for all $x, y \in R$. These relations lead to

$$\tau(x)(a\tau(y) - \tau(y)a) - (a\tau(y) - \tau(y)a)\tau(x) = 0$$

for all $x, y \in R$. Since τ is surjective, it follows that $a\tau(y) - \tau(y)a \in Z$. Hence, $a \in Z$ by Lemma 5 (ii).

Lemma 7. *Every Jordan τ-centralizer of R maps Z into Z.*

PROOF. Take an arbitrary $c \in Z$ and denote $a = f(c)$. Since f is a Jordan τ-centralizer of R, we have

$$2f(cx) = f(cx + xc) = f(c)\tau(x) + \tau(x)f(c) = a\tau(x) + \tau(x)a$$

for all $x \in R$. Put $g(x) = 2f(cx)$ for all $x \in R$. It is clear that g is also a Jordan τ-centralizer of R. By Lemma 6, we obtain $f(c) \in Z$ for all $c \in Z$, as required.

Lemma 8. *Let a and b be some fixed elements in R. If $a\tau(x) = \tau(x)b$ for all $x \in R$ then $a = b \in Z$.*

PROOF. By hypothesis, $a\tau(xy) = \tau(xy)b$ for all $x, y \in R$, whence $a\tau(x)\tau(y) = \tau(x)\tau(y)b = \tau(x)a\tau(y)$ for all $x, y \in R$. From the last relation we derive $(a\tau(x) - \tau(x)a)\tau(y) = 0$ for all $x, y \in R$. Since τ is surjective and R is semiprime, we have $a\tau(x) - \tau(x)a$ for all $x \in R$. Thus, $a \in Z$. By the hypothesis of the lemma, $(a - b)\tau(x) = 0$ for all $x \in R$. Hence, $a = b$.

Theorem 2. *If $\tau(Z) = Z$ then each Jordan τ-centralizer of R is a τ-centralizer.*

PROOF. Let f be a Jordan τ-centralizer of R, i.e.

$$f(xy + yx) = f(x)\tau(y) + \tau(y)f(x) = f(y)\tau(x) + \tau(x)f(y)$$

for all $x, y \in R$. Replacing y with $xy + yx$, we obtain

$$f(x)\tau(xy + yx) + \tau(xy + yx)f(x) = f(xy + yx)\tau(x) + \tau(x)f(xy + yx),$$
$$f(x)\tau(x)\tau(y) + \tau(y)\tau(x)f(x) = \tau(y)f(x)\tau(x) + \tau(x)f(x)\tau(y),$$
$$(f(x)\tau(x) - \tau(x)f(x))\tau(y) = \tau(y)(f(x)\tau(x) - \tau(x)f(x))$$

for all $x, y \in R$. Since τ is surjective, the last relation implies $f(x)\tau(x) - \tau(x)f(x) = [f(x), \tau(x)] \in Z$ for all $x \in R$. Our following aim is to show that $[f(x), \tau(x)] = 0$ holds for all $x \in R$. Take an arbitrary $c \in Z$. Since f is a Jordan τ-centralizer of R, we have

$$2f(cx) = f(cx + xc) = f(c)\tau(x) + \tau(x)f(c) = 2f(x)\tau(c)$$

for all $x \in R$ and $c \in Z$. By Lemma 7, $f(cx) = f(c)\tau(x) = f(x)\tau(c)$ for all $x \in R$ and $c \in Z$. Using the last relation and Lemma 7, we also obtain

$$[f(x), \tau(x)]\tau(c) = f(x)\tau(x)\tau(c) - \tau(x)f(x)\tau(c) = \tau(c)f(x)\tau(x) - \tau(x)f(x)\tau(c)$$
$$= f(c)\tau(x)\tau(x) - \tau(x)f(c)\tau(x) = f(c)\tau(x)\tau(x) - f(c)\tau(x)\tau(x) = 0$$

for all $x \in R$ and $c \in Z$. Hence, $[f(x), \tau(x)]\tau(c) = 0$ for all $x \in R$ and $c \in Z$. Since $\tau(Z) = Z$ by hypothesis and $[f(x), \tau(x)] \in Z$ for all $x \in R$, the previous equality implies that $[f(x), \tau(x)][f(x), \tau(x)] = 0$ for all $x \in R$. Since R is semiprime, we deduce $[f(x), \tau(x)] = 0$ for all $x \in R$. Thus, using the last relation, we obtain

$$2f(x^2) = f(xx + xx) = f(x)\tau(x) + \tau(x)f(x) = 2f(x)\tau(x)$$

for all $x \in R$. Since R is 2-torsion free; therefore, $f(x^2) = f(x)\tau(x)$ for all $x \in R$. It shows that f is a left Jordan τ-centralizer of R. Applying Theorem 1, we conclude the proof.

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