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Centralizing Mappings and Derivations in Prime Rings

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1. INTRODUCTION

Let R be a ring with center Z. A mapping F of R into itself is called centralizing on a subset S of R if $[s, F(s)] \in Z$ for all s in S; in the special case where $[s, F(s)] = 0$ for all s in S, the mapping F is said to be commuting on S. The classical result of Posner [23] states that the existence of a nonzero centralizing derivation on a prime ring forces the ring to be commutative. Mayne [19] proved the analogous result for centralizing automorphisms. A number of authors have extended these theorems of Posner and Mayne in several ways [1–6, 10–17, 20–22, 24]. In these papers the authors have shown that some concrete additive mappings (such as derivations, endomorphisms, etc.) cannot be centralizing on certain subsets of noncommutative prime (and some other) rings. The main purpose of this paper is to describe the structure of an arbitrary additive mapping which is centralizing on a prime ring. The result we shall prove is

THEOREM A. *Let R be a prime ring. Suppose an additive mapping F of R into itself is centralizing on R. If either R has a characteristic different from 2 or F is commuting on R, then F is of the form $F(x) = \lambda x + \zeta(x)$, $x \in R$, where λ is an element from the extended centroid C of R and ζ is an additive mapping of R into C.*

Theorem A is proved in Section 3. The proof depends on a result in Section 2, which gives a description of derivations d, g, and h of a prime ring R satisfying $d(x) = ag(x) + h(x)b$, $x \in R$, where a and b are some (fixed) elements in R. This result generalizes Theorem in Herstein's paper [9].

Our next aim is to initiate the study of a more general concept than centralizing mappings are; that is, we consider the situation when the mappings F and G of a ring R satisfy $F(s)s - sG(s) \in Z$ for all s in some subset

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S of R. This is also more general than the concept of skew-centralizing mappings: a mapping F of a ring R is said to be skew-centralizing on a subset S of R if $F(s)s + sF(s) \in Z$ for all $s \in S$. In this paper, following the most convenient way in the study of centralizing mappings, we consider the case when F and G are derivations. The goal of section 4 is to prove

THEOREM B. *Let R be a prime ring and U be a nonzero left ideal of R. Suppose derivations d and g of R satisfy $d(u)u - ug(u) \in Z$ for all $u \in U$. If $d \neq 0$ then R is commutative.*

Several authors [6, 10, 15, 21] have shown that if a prime ring R admits a nonzero derivation which is centralizing on some nonzero two-sided ideal U of R, then R is commutative. Bell and Martindale established this result under weaker hypothesis that U is a one-sided ideal [2, Thm. 4]. Theorem B is, of course, yet more general. Also, Theorem B generalizes a result in [10] asserting that the existence of a nonzero derivation which is skew-centralizing on some nonzero two-sided ideal in a prime ring implies that the ring is commutative.

Theorem B has also been inspired by the following observation. Let f be a generalized inner derivation of a ring R, i.e., $f(x) = ax + xb$ for some a, b in R. Note that the condition that f is centralizing on a subset S of R can be written in the form $[a, s]s - s[s, b] \in Z$ for all $s \in S$. Thus, introducing inner derivations d and g by $d(x) = [a, x]$ and $g(x) = [x, b]$, we obtain the same condition as in Theorem B, i.e., $d(s)s - sg(s) \in Z$ for all $s \in S$. Generalized inner derivations are extensively studied on operator algebras. Therefore, it might be interesting to investigate these mappings from an algebraical point of view.

Throughout, R will represent an associative ring with center Z; in case R is prime, by C we denote the extended centroid of R (see [18] or [8, pp. 20–31] for the notion of the extended centroid).

We shall make some use of the following well-known results.

Remarks. Let R be a prime ring.

1. The nonzero elements from Z are not zero divisors.
2. If d is a nonzero derivation of R then d does not vanish on a nonzero left ideal of R.
3. If R contains a commutative nonzero left ideal, then R is commutative.
4. Let c and ac be in the center of R. If c is not zero, then a is in the center of R.

We leave the proofs of these four easy facts to the reader. The following two results, the first due to Levitzki and the second due to Martindale, are deeper.

5. [7, Lemma 1.1]. R has no nonzero nil left ideals of bounded index.

6. [8, Lemma 1.3.2]. If a, $b \in R$ are such that $axb = bxa$ for all $x \in R$, and if $a \neq 0$, then $b = \lambda a$ for some λ in the extended centroid of R.

2. THE IDENTITY $d(x) = ag(x) + h(x)b$

The main purpose of this section is to prove the following theorem.

THEOREM 2.1. *Let R be a prime ring, and let d, g, h be derivations of R. Suppose there exist a, $b \in R$ such that*

$$d(x) = ag(x) + h(x)b \qquad \textit{for all} \quad x \in R. \tag{1}$$

If $a \notin Z$ and $b \notin Z$, then there exists $\lambda \in C$ such that $d(x) = [\lambda ab, x]$, $g(x) = [\lambda b, x]$ and $h(x) = [\lambda a, x]$ for all $x \in R$.

For the proof of Theorem 2.1 we need two lemmas which are of independent interest.

LEMMA 2.2. *Let R be a prime ring, and let d and g be derivations of R. Suppose that*

$$d(x)\, g(y) = g(x)\, d(y) \qquad \textit{for all} \quad x, y \in R. \tag{2}$$

If $d \neq 0$ then there exists $\lambda \in C$ such that $g(x) = \lambda d(x)$ for all $x \in R$.

Proof. Replacing y by yz in (2), we get $d(x)\, g(y)z + d(x)\, yg(z) = g(x)\, d(y)z + g(x)\, yd(z)$. According to (2) this relation reduces to

$$d(x)\, yg(z) = g(x)\, yd(z) \qquad \text{for all} \quad x, y, z \in R. \tag{3}$$

In particular, $d(x)\, yg(x) = g(x)\, yd(x)$ for all $x, y \in R$. Hence, if $d(x) \neq 0$, using Remark 6 we have that $g(x) = \lambda(x)\, d(x)$ for some $\lambda(x) \in C$. Thus, if $d(x) \neq 0$ and $d(z) \neq 0$, then it follows from (3) that

$$(\lambda(x) - \lambda(z))\, d(x)\, yd(z) = 0$$

for all $y \in R$. Since R is prime this relation implies that $\lambda(x) = \lambda(z)$. Thus we have proved that there exists $\lambda \in C$ such that the relation $g(x) = \lambda d(x)$ holds for all $x \in R$ with the property $d(x) \neq 0$. On the other hand, if $d(x) = 0$ then we see from (3), since $d \neq 0$ and R is prime, that $g(x) = 0$ as well. Thus $g(x) = \lambda d(x)$ for all $x \in R$.

LEMMA 2.3. *Let R be a prime ring, and let d, f, g and h be derivations of R. Suppose that*

$$d(x)\, g(y) = h(x)\, f(y) \qquad \textit{for all} \quad x, y \in R. \tag{4}$$

If $d \neq 0$ *and* $f \neq 0$ *then there exists* $\lambda \in C$ *such that* $g(x) = \lambda f(x)$ *and* $h(x) = \lambda d(x)$ *for all* $x \in R$.

Proof. Taking $y = zy$ in (4) we obtain $d(x)\,g(z)\,y + d(x)\,zg(y) = h(x)\,f(z)\,y + h(x)\,zf(y)$; applying (4) we then get

$$d(x)\,zg(y) = h(x)\,zf(y) \qquad \text{for all} \quad x, y, z \in R. \tag{5}$$

Letting $z = zf(w)$ in (5) we get $d(x)\,zf(w)\,g(y) = h(x)\,zf(w)\,f(y)$. By (5), $h(x)\,zf(w) = d(x)\,zg(w)$ and so we have $d(x)\,z(f(w)\,g(y) - g(w)\,f(y)) = 0$. Since $d \neq 0$ and R is prime this relation implies $f(w)\,g(y) = g(w)\,f(y)$ for all $y, w \in R$. We have assumed that $f \neq 0$, hence it follows from Lemma 2.2 that $g(y) = \lambda f(y)$ for all $y \in R$, where λ is an element in C. Hence (5) becomes $(\lambda d(x) - h(x))\,zf(y) = 0$ for all $x, y, z \in R$. Consequently $h(x) = \lambda d(x)$ for all $x \in R$.

Proof of Theorem 2.1. According to (1) we have

$$\begin{aligned} &ag(x)\,y + h(x)\,by + xag(y) + xh(y)b \\ &\quad = d(x)\,y + xd(y) = d(xy) = ag(xy) + h(xy)b \\ &\quad = ag(x)\,y + axg(y) + h(x)\,yb + xh(y)b. \end{aligned}$$

Hence $[a, x]\,g(y) = h(x)[b, y]$ for all $x, y \in R$. By Lemma 2.3 there exists $\lambda \in C$ such that $h(x) = [\lambda a, x]$ and $g(x) = [\lambda b, x]$ for all $x \in R$. Hence (1) yields $d(x) = [\lambda ab, x]$ for all $x \in R$.

In [9], I. N. Herstein proved the following result: If a derivation $d \neq 0$ of a prime ring R and an element $a \notin Z$ are such that $[a, d(x)] = 0$ for all $x \in R$, then R has a characteristic 2, $a^2 \in Z$ and $d(x) = [\lambda a, x]$, where $\lambda \in C$, for all $x \in R$. We are now in a position to generalize Herstein's result.

COROLLARY 2.4. *Let* R *be a prime ring, and let* g *and* h *be derivations of* R. *Suppose there exist* $a, b \in R$ *such that* $ag(x) + h(x)b = 0$ *for all* $x \in R$. *If* $a \notin Z$ *and* $b \notin Z$, *then there exists* $\lambda \in C$ *such that* $g(x) = [\lambda b, x]$ *and* $h(x) = [\lambda a, x]$ *for all* $x \in R$. *Moreover, if* $g \neq 0$ *then* $ab \in Z$.

Proof. The first part follows immediately from Theorem 2.1. If $g \neq 0$ then $\lambda \neq 0$ and so $ag(x) + h(x)b = 0$ implies $ab \in Z$.

3. CENTRALIZING MAPPINGS OF PRIME RINGS

First we show that under rather weak hypothesis every centralizing mapping is in fact commuting.

PROPOSITION 3.1. *Let R be a 2-torsion free semiprime ring and U be a Jordan subring of R. If an additive mapping F of R into itself is centralizing on U, then F is commuting on U.*

Proof. A linearization of $[F(x), x] \in Z$ gives $[F(x), y] + [F(y), x] \in Z$ for all $x, y \in U$. In particular, $[F(x), x^2] + [F(x^2), x] \in Z$. Since $[F(x), x] \in Z$ we have $[F(x), x^2] = 2[F(x), x]x$. Thus

$$2[F(x), x]x + [F(x^2), x] \in Z \qquad \text{for all} \quad x \in U. \tag{6}$$

By assumption, $[F(x^2), x^2] \in Z$ for all $x \in U$. That is,

$$[F(x^2), x]x + x[F(x^2), x] \in Z \qquad \text{for all} \quad x \in U. \tag{7}$$

Now fix $x \in U$ and let $z = [F(x), x] \in Z$, $u = [F(x^2), x]$. We must show that $z = 0$. By (6) we have $0 = [F(x), 2zx + u] = 2z^2 + [F(x), u]$. Thus

$$[F(x), u] = -2z^2. \tag{8}$$

According to (7) we have $0 = [F(x), ux + xu] = [F(x), u]x + u[F(x), x] + [F(x), x]u + x[F(x), u]$; applying (8) we then get $-4z^2x + 2zu = 0$. Thus $zu = 2z^2x$. Multiplying (8) by z and using the last relation we obtain $-2z^3 = [F(x), 2z^2x] = 2z^3$. Hence $z^3 = 0$. Since the center of a semiprime ring contains no nonzero nilpotents, we conclude that $z = 0$. This proves the proposition.

We come now to the main result of this paper.

THEOREM 3.2. *Let R be a prime ring. If an additive mapping F of R is commuting on R, then there exist $\lambda \in C$ and an additive mapping $\xi: R \to C$, such that $F(x) = \lambda x + \xi(x)$ for all $x \in R$.*

Proof. Linearizing $[x, F(x)] = 0$ we get $[x, F(y)] = [F(x), y]$ for all $x, y \in R$. Hence $[x, F(yz)] = [F(x), yz] = y[F(x), z] + [F(x), y]z = y[x, F(z)] + [x, F(y)]z$. Thus we have

$$[x, F(yz)] = y[x, F(z)] + [x, F(y)]z \qquad \text{for all} \quad x, y, z \in R. \tag{9}$$

This is the key identity, as we shall see. Fix $y \in R$. Suppose $y \notin Z$. As a special case of (9) we have $[x, F(y^2)] = y[x, F(y)] + [x, F(y)]y$ for all $x \in R$. Since the mappings $x \to [x, F(y^2)]$ and $x \to [x, F(y)]$ are derivations, Theorem 2.1 can be applied. Thus there exists $\lambda(y) \in C$ such that $[x, F(y)] = [x, \lambda(y)y]$ for all $x \in R$. Now, suppose $y \in Z$. From the linearized form of $[F(x), x] = 0$ we see that $F(y) \in Z$ as well. It is now clear that for every $y \in R$ there exists $\lambda(y) \in C$ such that $[x, F(y)] = [x, \lambda(y)y]$ is fulfilled for any $x \in R$. We want to show that $\lambda(y)$ is a constant.

Now, (9) can be written in the form $[x, \lambda(yz)yz] = y[x, \lambda(z)z] + [x, \lambda(y)y]z$; that is,

$$[x, (\lambda(yz) - \lambda(y))y]z + y[x, (\lambda(yz) - \lambda(z))z] = 0. \tag{10}$$

Take $y \notin Z$, $z \notin Z$. By (10) and Theorem 2.1 it follows that there exists $\mu \in C$ such that $[x, (\lambda(yz) - \lambda(y))y] = [x, \mu y]$ and $[x, (\lambda(yz) - \lambda(z))z] = [x, \mu z]$ for all $x \in R$. Since $y \notin Z$ and $z \notin Z$ these relations imply that $\lambda(yz) - \lambda(y) = \mu$ and $\lambda(yz) - \lambda(z) = \mu$. Consequently $\lambda(y) = \lambda(z)$. Thus there exists $\lambda \in C$ such that $[x, F(y)] = [x, \lambda y]$ holds for all $x \in R$ and $y \notin Z$. However, since F maps Z into itself, this relation is certainly true if $y \in Z$. Finally, note that the mapping $\xi(y) = F(y) - \lambda y$ has the desired properties. The proof the theorem is thereby complete.

Combining Proposition 3.1 and Theorem 3.2 we obtain Theorem A.

Remark 3.3. In [22], C. R. Miers studied centralizing mappings on C^*-algebras. He showed, that if A is a C^*-algebra, p is a complex polynomial and d is a derivation of A such that $p(d)$ is commuting on A, then $p(d) = 0$ [22, Thm. 1]. Using Theorem 3.2, a similar result can be easily obtained for inner derivations in semiprime rings (we remark that Miers had first considered the case where A is a von Neumann algebra and so d is an inner derivation). Let R be a semiprime ring, a be an element in R and d_a be the inner derivation $d_a(x) = [a, x]$. Suppose that the mapping $F\colon R \to R$,

$$F(x) = c_1 d_a(x) + c_2 d_a^2(x) + \cdots + c_n d_a^n(x), \qquad x \in R,$$

where $c_1, c_2, ..., c_n$ are elements in R, is commuting on R. We intend to show that $F = 0$. First assume R is prime. By Theorem 3.2 we have $F(x) = \lambda x + \xi(x)$. Since $F(a) = 0$, we then have $\lambda a = -\xi(a)$. From this relation it follows at once that if $\lambda \neq 0$ then $a \in Z$. Therefore we may assume that $\lambda = 0$. Note that for every $x \in R$, $F(xa) = F(x)a$. Since F maps R into C it follows that either $a \in Z$ or $F(x) = 0$ for all $x \in R$; in any case $F = 0$. Now, let R be semiprime. Choose an arbitrary prime ideal P of R. A mapping F may be dropped to a mapping F_P on R/P. Then F_P is commuting on R/P and by the above argument $F_P = 0$. By the semiprimeness of R, we conclude that $F = 0$ as well.

4. THE CASE $d(u)u - ug(u) \in Z$

THEOREM 4.1. *Let R be a prime ring and U be a nonzero left ideal of R. Suppose that derivations d and g of R are such that $d(u)u - ug(u) \in Z$ for all $u \in U$. If $d \neq 0$ then R is commutative.*

If we assume that $g \neq 0$ instead of $d \neq 0$ then the result need not be true. Indeed, let R be any prime ring having nilpotent elements, and let $a \neq 0 \in R$ be such that $a^2 = 0$. Let U be a left ideal generated by a. Define the inner derivation g by $g(x) = [a, x]$. Then $Ug(u) = 0$ for all $u \in U$.

For the proof of Theorem 4.1 we need the following lemma, which is in fact a very special case of Theorem 4 in [2]. However, we present the proof since it is rather short.

LEMMA 4.2. *Let R be a noncommutative prime ring and U be a nonzero left ideal of R. If a derivation d of R maps U into the center of R, then $d = 0$.*

Proof. Take u, $v \in U$. Then $d(u)$, $d(v)$ and $d(vu)$ are contained in Z. Hence $0 = [d(vu), u] = [d(v)u + vd(u), u] = [v, u]\, d(u)$. From Remark 1 it follows that either $d(u) = 0$ or u is contained in the center of U. In other words, U is the union of its subsets $G = \{u \in U \mid d(u) = 0\}$ and $H = \{u \in U \mid u \text{ is contained in the center of } U\}$; note that both are additive subgroups of U. But a group cannot be the union of two proper subgroups. Thus either $G = U$ or $H = U$. If $H = U$ then U is commutative which is impossible by Remark 3. Hence $G = U$ and using Remark 2 we obtain the assertion of the lemma.

Proof of Theorem 4.1. We assume that R is a noncommutative prime ring, and d, g are derivations of R such that $d(u)u - ug(u) \in Z$ for all u in a nonzero left ideal U. We want to show that $d = 0$.

A linearization of $d(u)u - ug(u) \in Z$ gives

$$d(u)v + d(v)u - ug(v) - vg(u) \in Z \qquad \text{for all} \quad u, v \in U. \tag{11}$$

First assume there exists $c \neq 0 \in Z \cap U$. Taking $v = c$ in (11) we get

$$c(d(u) - g(u)) + (d(c) - g(c))u \in Z \qquad \text{for all} \quad u \in U. \tag{12}$$

Now let $v = c^2$ in (11). Then we obtain $c^2(d(u) - g(u)) + 2c(d(c) - g(c))u \in Z$. That is,

$$c\{c(d(u) - g(u)) + (d(c) - g(c))u\} + c(d(c) - g(c))u \in Z.$$

Noting that the first summand is contained in Z by (12), we get $c(d(c) - g(c))u \in Z$ for every u in U. By Remark 3 there exists $u \in U$ which is not contained in Z, hence it follows from the last relation, Remark 4 and Remark 1 that $d(c) = g(c)$. Thus (12) becomes $c(d(u) - g(u)) \in Z$ for all $u \in U$, and so, by Remark 4, $d(u) - g(u) \in Z$ for every u in U. In view of Lemma 4.2 we are forced to conclude that $d = g$. Now apply [2, Lemma 4]. Thus, in case $Z \cap U \neq 0$ we have $d(u)u = ug(u)$ for all $u \in U$.

Now assume $Z \cap U = 0$. By assumption, $d(u)u - ug(u) \in Z$ for $u \in U$, so this commutes with any $v \in U$ and shows that $vug(u) \in U$. A linearization gives $vug(w) + vwg(u) \in U$. Replacing w by vu we get $vug(vu) \in U$. Choose $u \in U$ such that $W = Uu \neq 0$; for $w \in W$ we then have $d(w)w - wg(w) \in U \cap Z = 0$.

Thus we have proved that in any case there exists a nonzero left ideal, which we denote by W, such that $d(w)w = wg(w)$ for all $w \in W$. Linearizing this relation we obtain

$$d(u)w + d(w)u = ug(w) + wg(u) \qquad \text{for all} \quad u, w \in W. \tag{13}$$

Replace in (13) w by wu. The relation which we obtain can be written in the form

$$(d(u)w + d(w)u - ug(w))u + w(d(u)u - ug(u)) = uwg(u);$$

hence it follows from (13) and $d(u)u = ug(u)$ that

$$wg(u)u = uwg(u) \qquad \text{for all} \quad u, w \in W. \tag{14}$$

Replacing w by vw and applying (14), we then get $[v, u]\, wg(u) = 0$. Thus, $[W, u]\, RWg(u) = 0$ for all $u \in W$. Since R is prime, for every $u \in W$ we have either $[W, u] = 0$ or $Wg(u) = 0$. The subsets $A = \{u \in W \mid [W, u] = 0\}$ and $B = \{u \in W \mid Wg(u) = 0\}$ are additive subgroups of W and by the above, their union is equal to W. Therefore either $A = W$ or $B = W$. If $A = W$ then R is commutative by Remark 3. Hence $B = W$. In particular, $ug(u) = 0$ for all $u \in W$, which yields

$$d(u)u = 0 \qquad \text{for all} \quad u \in W. \tag{15}$$

Linearizing (15) we get

$$d(u)v + d(v)u = 0 \qquad \text{for all} \quad u, v \in W. \tag{16}$$

Replace v by $d(u)v$ to get $0 = d(u)^2 v + d^2(u)\, vu + d(u)\, d(v)u = d^2(u)\, vu$, since $d(u)v = -d(v)u$. Thus $d^2(u)\, RWu = 0$ for all $u \in W$. Using primeness of R and the fact that a group cannot be the union of two proper subgroups, it follows that $d^2(u) = 0$ for all $u \in W$. According to (15) we then have $0 = d(d(u)u) = d^2(u)u + d(u)^2$, which yields $d(u)\, d(v) + d(v)\, d(u) = 0$ for $u, v \in W$. Note that the last relation implies $d(v)\, d(W)\, d(W)v = 0$, and also that $d(u)\, d(v)\, d(u) = 0$. In the latter expression, replace v by wv, right multiply by $d(w)v$, and use the last sentence to conclude that $(d(u)\, d(w)v)^2 = 0$. This means that $Wd(u)\, d(w)$ is a nil left ideal of index three, which is impossible by Remark 5 unless $Wd(u)\, d(w) = 0$. Replacing u by uv, one shows that $Wd(u)\, vd(w) = 0$; since R is prime, we then have $Wd(W) = 0$. Next, by (16) we have $d(u)(uv) + d(uv)u = 0$, hence $d(uv)u = 0$

by (15), and therefore $d(u)vu=0$ since $Wd(W)=0$. Thus $d(u)RWu=0$ for all $u \in W$, from which one concludes easily that $d(W)=0$. But then $d=0$ (Remark 2). The proof of the theorem is complete.

We conclude the paper with some corollaries of Theorem 4.1, which were outlined at the beginning of this paper.

COROLLARY 4.3. *Let R be a prime ring and U be a nonzero left ideal of R. If there exists a nonzero derivation of R which is centralizing or skew-centralizing on U, then R is commutative.*

COROLLARY 4.4. *Let R be a noncommutative prime ring and U be a nonzero left ideal of R. Suppose there exist $a, b \in R$ and a derivation d of R such that the mapping $x \to d(x)+ax+xb$ is centralizing on U. Then d is an inner derivation given by $d(x)=[x, a]$.*

Proof. Observe that the relation $[d(u)+au+ub, u] \in Z$ *can be written in the form* $(d(u)-[u, a])u-u(d(u)-[b, u]) \in Z$.

Taking a derivation d in Corollary 4.4 to be zero we get

COROLLARY 4.5. *Let R be a prime ring and U be a nonzero left ideal of R. If $a, b \in R$ are such that the mapping $x \to ax+xb$ is centralizing on U, then $a \in Z$.*

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