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HIGHER DERIVATIONS AND A THEOREM BY HERSTEIN

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Abstract. In this paper we extend to the higher derivations a well-known result proved by Herstein concerning derivations in prime rings. We prove results which imply that every Jordan higher derivation of a 2-torsion-free semiprime ring is a higher derivation.

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0. Introduction. Let R be a ring not necessarily with identity element. A derivation (resp. Jordan derivation) d of R is an additive mapping $d : R \to R$ such that $d(ab) = d(a)b + ad(b)$, for every $a, b \in R$ (resp. $d(a^2) = d(a)a + ad(a)$, for every $a \in R$). Every derivation is obviously a Jordan derivation and the converse is in general not true. In 1957 Herstein proved that if R is a prime ring of characteristic different of 2, then every Jordan derivation of R is a derivation ([9], Theorem 3.1). Later on M. Brešar [3] extended the result to 2-torsion-free semiprime rings. In a subsequent paper Brešar gave another proof of this result using Jordan triple derivations. An additive mapping $d : R \to R$ is said to be a Jordan triple derivation if $d(aba) = d(a)ba + ad(b)a + abd(a)$, for every $a, b \in R$. He proved that every Jordan triple derivation of a 2-torsion-free semiprime ring is a derivation ([4], Theorem 4.3). It turns out that every Jordan derivation of a 2-torsion-free ring is a Jordan triple derivation ([10], Lemma 3.5). This gives another proof of the result of Herstein for 2-torsion-free semiprime rings.

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Also, R. Awtar extended the Herstein's Theorem to Lie ideals ([1], Theorem). He proved that if U is a Lie ideal of a prime ring R of characteristic different of 2 such that $u^2 \in U$, for every $u \in U$, and $d : R \to R$ is an additive mapping such that $d\,|_U$ is a Jordan derivation of U into R, then $d\,|_U$ is a derivation of U into R.

On the other hand, higher derivations have been studied in many papers mainly in commutative rings, but also in non-commutative rings ([5], [6], [7]). The purpose of this paper is to extend the above results to higher derivations. We give the corresponding natural definitions and we prove the results extending the above, as we state more precisely in the next section. Finally, we point out that our proofs here are unified proofs for both cases, while the former ones are quite different.

1. Definitions and main results. Throughout this paper R is a ring with center $Z(R)$ and U is a Lie ideal of R. Also, $\mathbb{N}$ denotes the set of natural numbers including 0 and $D = (d_i)_{i\in\mathbb{N}}$ is a family of additive mappings of R such that $d_0 = id_R$.

In the main result of this paper we assume that the Lie ideal U is not contained in $Z(R)$ and $u^2 \in U$, for every $u \in U$. A Lie ideal of this type will be called an admissible Lie ideal. We begin with the following

DEFINITION 1.1. D is said to be:

a higher derivation (HD for short) if for every $n \in \mathbb{N}$ we have $d_n(ab) = \sum_{i+j=n} d_i(a)d_j(b)$, for all $a, b \in R$ ([11], Exerc. 4, p. 540);

a Jordan higher derivation (JHD for short) if for every $n \in \mathbb{N}$ we have $d_n(a^2) = \sum_{i+j=n} d_i(a)d_j(a)$, for all $a \in R$;

a Jordan triple higher derivation (JTHD for short) if for every $n \in \mathbb{N}$ we have $d_n(aba) = \sum_{i+j+k=n} d_i(a)d_j(b)d_k(a)$, for all $a, b \in R$.

Similarly, if U is a Lie ideal of R, then D is said to be *a HD (JHD, JTHD) of U into R* in case that the above corresponding conditions are satisfied for all $a, b \in U$.

We prove the following:

THEOREM 1.2. *Let R be a 2-torsion-free semiprime ring (resp. prime ring of characteristic different of 2 and U an admissible Lie ideal of R). Then every JTHD of R (resp. of U into R) is a HD of R (resp. of U into R).*

THEOREM 1.3. *Let R be a 2-torsion-free ring and U a Lie ideal of R. Then every JHD of R (resp. of U into R) is a JTHD of R (resp. of U into R).*

The following is clear:

COROLLARY 1.4. *Let R be a 2-torsion-free semiprime ring (resp. prime ring of characteristic different of 2 and U an admissible Lie ideal of R). Then every JHD of R (resp. of U into R) is a HD of R (resp. of U into R).*

One can ask whether the result of Corollary 1.4 for Lie ideals is also true in the semiprime case. We include an example by M. Brešar showing that without some additional assumption this is not the case.

2. Prerequisites. The following well-known result will be used in the paper ([2], Lemma 4).

LEMMA 2.1. *Let R be a prime ring of characteristic different of 2 and U a Lie ideal of R with $U \not\subset Z(R)$. If $a, b \in R$ and $aUb = 0$, then either $a = 0$ or $b = 0$.*

Assume that U is a Lie ideal satisfying the condition $u^2 \in U$, for every $u \in U$. For $u, v \in U$, $(uv + vu) = (u + v)^2 - (u^2 + v^2)$ and so $uv + vu \in U$. Also, $[u, v] = uv - vu \in U$ and it follows that $2uv \in U$. Hence $4uvw = 2(2uv)w \in U$, for every $u, v, w \in U$. This remark will be used freely in the rest of the paper.

The following lemmas can be found in ([4], Lemmas 1.1 and 1.2), but we present here an extension of them.

LEMMA 2.2. *Let R be a 2-torsion-free semiprime ring (resp. prime ring) and U an admissible Lie ideal of R. If $a, b \in R$ (resp. $a \in U$ and $b \in R$) are such that $axb + bxa = 0$, for every $x \in R$ (resp. $x \in U$), then $axb = bxa = 0$ for every $x \in R$ (resp. $a = 0$ or $b = 0$).*

Proof. We just prove the second case following ([4], Lemma 1.1). Take $x, y \in U$ and use three times the relation $arb = -bra$, for $r \in U$. We obtain $4axbyaxb = -b(4xay)a(xb) = 4axaybxb = -4axbyaxb$. Thus $axbyaxb = 0$, for every $x, y \in U$, and Lemma 2.1 completes the proof. ∎

LEMMA 2.3. *Assume that R is 2-torsion-free and semiprime (resp. prime) and U is an admissible Lie ideal of R. Let $G_1, G_2, \ldots, G_n$ be additive groups, $S : G_1 \times \ldots \times G_n \to R$ and $T : G_1 \times \ldots \times G_n \to R$ mappings which are additive in each argument. If $S(a_1, \ldots, a_n)xT(a_1, \ldots, a_n) = 0$, for every $x \in R$ (resp. $x \in U$), $a_i \in G_i$, $i = 1, 2, \ldots, n$, then $S(a_1, \ldots, a_n)xT(b_1, \ldots, b_n) = 0$, for every $x \in R$, $a_i, b_i \in G_i$, $i = 1, 2, \ldots, n$ (resp. $S(a_1, \ldots, a_n) = 0$, for every $a_i \in G_i$, $i = 1, 2, \ldots, n$, or $T(b_1, \ldots, b_n) = 0$, for every $b_i \in G_i$, $i = 1, 2, \ldots, n$).*

Proof. As in ([4], Lemma 1.2) it suffices to prove the case $n = 1$. We prove the second case. If $S(a)xT(a) = 0$, for every $x \in U$, $a \in G_1$ we have that $(T(a)xS(a))y(T(a)xS(a)) = 0$, for $x, y \in U$ and by Lemma 2.1 $T(a)xS(a) = 0$, for every $x \in U$, $a \in G_1$. Linearizing $S(a)xT(a) = 0$ we obtain $S(a)xT(b) + S(b)xT(a) = 0$, for every $x \in U$, $a, b \in G_1$. Hence $(S(a)xT(b))y(S(a)xT(b)) = -S(a)xT(b)yS(b)xT(a) = 0$, for every $x, y \in U$. The result follows applying Lemma 2.1. ∎

Remark. As the referee pointed out in Lemmas 2.2 and 2.3 the assumption for a Lie ideal U to be admissible is stronger than needed. His (her) proofs show that the results remain true provided that U is a non-central Lie ideal. For example, Lemma 2.2 can be stated as follows: Let R be a 2-torsion-free prime ring and U a non-central Lie ideal of R. If $a, b \in R$ are such that $axb + bxa = 0$, for every $x \in U$, then either $a = 0$ or $b = 0$. Since the above results suffice for our needs these proofs will not be included here.

For elements $a, b, c \in R$ we put $[a, b, c] = abc - cba$.

Lemma 2.4. *Assume that R is prime of characteristic different of 2 and $U \not\subset Z(R)$. Then there exist elements $a, b, c \in U$ such that $[a, b, c] \neq 0$.*

Proof. Assume that $[x, y, z] = 0$, for every $x, y, z \in U$. By ([12], Lemma 1) there exists $u \in U$ with $u^2 \neq 0$. Also $[u^2, v] = [u, u, v] = 0$, for every $v \in U$, and so by ([12], Lemma 8) we have that $u^2 \in Z(R)$. Thus the relation $[x, u^2, z] = 0$ gives $[U, U] = 0$, which contradicts ([12], Lemma 7). ■

3. Proofs. To prove Theorem 1.2 we need several lemmas. Throughout this section R is a ring, U is a Lie ideal of R and $D = (d_i)_{i \in \mathbb{N}}$ a JTHD of R (resp. of U into R).

We denote by $\varphi_n(a, b, c)$ the element of R defined by

$$\varphi_n(a, b, c) \doteq d_n(abc) - \sum_{i+j+k=n} d_i(a) d_j(b) d_k(c).$$

Then φ_n is additive in each argument and $\varphi_n(a, b, a) = 0$, for all $a, b \in R$.

Lemma 3.1. *For every $a, b, c \in R$ (resp. $a, b, c \in U$) and every $n \in \mathbb{N}$, we have*

$$d_n(abc + cba) = \sum_{i+j+k=n} (d_i(a) d_j(b) d_k(c) + d_i(c) d_j(b) d_k(a)).$$

Proof. It follows immediately from the relation $\varphi_n(a + c, b, a + c) = 0$. ■

Lemma 3.2. *For every $a, b, c \in R$ (resp. $a, b, c \in U$) and $n \in \mathbb{N}$, we have:*

(i) $\varphi_n(c, b, a) = -\varphi_n(a, b, c)$.

(ii) $2\varphi_n(a, b, c) = d_n([a, b, c]) + \sum_{i+j+k=n} [d_i(c), d_j(b), d_k(a)]$.

Proof. (i) It follows easily from Lemma 3.1.

(ii) Applying (i) we have

$$2\varphi_n(a, b, c) = \varphi_n(a, b, c) - \varphi_n(c, b, a) =$$

$$d_n(abc) - \sum_{i+j+k=n} d_i(a) d_j(b) d_k(c) - d_n(cba) + \sum_{i+j+k=n} d_i(c) d_j(b) d_k(a)$$

and (ii) follows. ■

Lemma 3.3. *Let $n \in \mathbb{N}$. If $\varphi_m(a, b, c) = 0$ for every $m < n$, $a, b, c \in R$ (resp. $a, b, c \in U$ and R is 2-torsion-free) then*

$$\varphi_n(a, b, c) r [a, b, c] + [a, b, c] r \varphi_n(a, b, c) = 0,$$

for every $r, a, b, c \in R$ (resp. $r, a, b, c \in U$).

Proof. First we assume that D is a JTHD of R. Take $a, b, c, r \in R$ and put $w = abcrcba + cbarabc$. By definition of JTHD we have

$$\begin{aligned}
&d_n(w) = d_n(a(bcrcb)a + c(barab)c) = \\
&\sum_{i+j+k=n} (d_i(a)d_j(b(crc)b)d_k(a) + d_i(c)d_j(b(ara)b)d_k(c)) = \\
&\sum_{i+j+k=n} d_i(a) \sum_{l+t+u=j} d_l(b)d_t(crc)d_u(b)d_k(a) + \\
&\sum_{i+j+k=n} d_i(c) \sum_{l+t+u=j} d_l(b)d_t(ara)d_u(b)d_k(c) = \\
&\sum_{i+j+k=n} d_i(a) \sum_{l+t+u=j} d_l(b) \sum_{s+p+q=t} d_s(c)d_p(r)d_q(c)d_u(b)d_k(a) + \\
&\sum_{i+j+k=n} d_i(c) \sum_{l+t+u=j} d_l(b) \sum_{s+p+q=t} d_s(a)d_p(r)d_q(a)d_u(b)d_k(c) = \\
&\sum_{\sigma=n} d_i(a)d_l(b)d_s(c)d_p(r)d_q(c)d_u(b)d_k(a) + \\
&\sum_{\sigma=n} d_i(c)d_l(b)d_s(a)d_p(r)d_q(a)d_u(b)d_k(c),
\end{aligned}$$

where $\sigma = i + l + s + p + q + u + k$.

On the other hand Lemma 3.1 implies that

$$\begin{aligned}
&d_n(w) = d_n((abc)r(cba) + (cba)r(abc)) = \\
&\sum_{\alpha+\beta+\gamma=n} (d_\alpha(abc)d_\beta(r)d_\gamma(cba) + d_\alpha(cba)d_\beta(r)d_\gamma(abc)).
\end{aligned}$$

By assumption, in the expression $x = \sum_{\alpha+\beta+\gamma=n}(d_\alpha(abc)d_\beta(r)d_\gamma(cba))$ we can substitute $d_\alpha(abc)$ for $\sum_{i+j+l=\alpha} d_i(a)d_j(b)d_l(c)$, when $\alpha < n$. Similarly for $d_\gamma(cba)$, when $\gamma < n$. Using this we can easily see that

$$x - \sum_{\sigma=n}(d_i(a)d_l(b)d_s(c)d_p(r)d_q(c)d_u(b)d_k(a)) = \varphi_n(a, b, c)rcba + abcr\varphi_n(c, b, a),$$

where σ is as above. The last relation also implies that

$$y - \sum_{\sigma=n}(d_i(c)d_l(b)d_s(a)d_p(r)d_q(a)d_u(b)d_k(c)) = \varphi_n(c, b, a)rabc + cbar\varphi_n(a, b, c),$$

where $y = \sum_{\alpha+\beta+\gamma=n}(d_\alpha(cba)d_\beta(r)d_\gamma(abc))$. It follows that

$$\varphi_n(a, b, c)rcba + abcr\varphi_n(c, b, a) + \varphi_n(c, b, a)rabc + cbar\varphi_n(a, b, c) = 0.$$

Finally, using Lemma 3.2, (i), it is easy to complete the proof of the first part.

Now, assume that D is a JTHD of U into R and take $a, b, c, r \in U$. Define w as above. Thus $2^4 w = 2^4(abcrcba + cbarabc) = a(4b(4crc)b)a + c(4b(4ara)b)c = (4abc)r(4cba) + (4cba)r(4abc)$, where $4crc, 4b(4crc)b, 4ara, 4b(4ara)b, 4abc, 4cba$ are in U. Since R is 2-torsion-free, the relations used in the first part remain valid. Hence the proof is the same. ∎

LEMMA 3.4. *Assume that R a is 2-torsion-free semiprime ring, D is JTHD of R and $n \in \mathbb{N}$. If $\varphi_m(a,b,c) = 0$, for every $a,b,c \in R$ and $m < n$, then $\varphi_n(a,b,c)rd_k([a,b,c]) = 0$ and $d_k([a,b,c])r\varphi_n(a,b,c) = 0$, for every $k < n$ and $a,b,c,r \in R$.*

Proof. We prove the first relation and the second one can be proved similarly. By Lemmas 3.3, 2.2 and 2.3 we have

$$\varphi_n(a_1,b_1,c_1)r[a_2,b_2,c_2] = 0, \tag{1}$$

for all $r, a_i, b_i, c_i \in R$, $i = 1,2$, and the result holds for $k = 0$. Now

$$\varphi_n(a,b,c)rd_k([a,b,c]) = \varphi_n(a,b,c)rd_k(abc) - \varphi_n(a,b,c)rd_k(cba).$$

By assumption, $d_k(abc) = \sum_{i+j+l=k} d_i(a)d_j(b)d_l(c)$, for every $a,b,c \in R$, $k < n$, and we have a similar relation for $d_k(cba)$. Consequently

$$\varphi_n(a,b,c)rd_k([a,b,c]) = \varphi_n(a,b,c)r \sum_{i+j+l=k} [d_i(a), d_j(b), d_l(c)] = 0,$$

by (1). The proof is complete. ■

LEMMA 3.5. *Assume that R is a 2-torsion-free semiprime ring (resp. prime ring and U an admissible Lie ideal of R). Then $\varphi_n(a,b,c) = 0$, for every $a,b,c \in R$ (resp. $a,b,c \in U$), $n \in \mathbb{N}$.*

Proof. Assume that D is a JTHD of R. By definition, for any $a,b,c \in R$ we have $\varphi_0(a,b,c) = 0$. Assume, by induction, that $\varphi_m(a,b,c) = 0$, i.e., $d_m(abc) = \sum_{i+j+k=m} d_i(a)d_j(b)d_k(c)$, for every $a,b,c \in R$ and $m < n$.

Take elements $a,b,c,r \in R$. By Lemma 3.3

$$d_n(\varphi_n(a,b,c)r[a,b,c] + [a,b,c]r\varphi_n(a,b,c)) = 0.$$

We develop this relation using Lemma 3.1, we left multiply it by $\varphi_n(a,b,c)s$ and right by $s\varphi_n(a,b,c)$, $s \in R$, and apply Lemma 3.4. We obtain

$$\varphi_n(a,b,c)s\varphi_n(a,b,c)rd_n([a,b,c])s\varphi_n(a,b,c)+$$

$$\varphi_n(a,b,c)sd_n([a,b,c])r\varphi_n(a,b,c)s\varphi_n(a,b,c) = 0.$$

Note that by Lemma 3.2, (ii), and the relation (1) we have

$$2\varphi_n(a,b,c)r\varphi_n(a,b,c) = d_n([a,b,c])r\varphi_n(a,b,c)$$

and

$$2\varphi_n(a,b,c)r\varphi_n(a,b,c) = \varphi_n(a,b,c)rd_n([a,b,c]).$$

Combining the last three relations we obtain

$$4\varphi_n(a,b,c)s\varphi_n(a,b,c)r\varphi_n(a,b,c)s\varphi_n(a,b,c) = 0, \text{ for every } r,s \in R.$$

Since R is semiprime and 2-torsion free the result follows in this case.

Now, assume that R is prime, U is admissible and D is a JTHD of U into R. As above, for any $a,b,c \in U$ we have $\varphi_0(a,b,c) = 0$. We assume, by induction, that $\varphi_m(a,b,c) = 0$, for every $a,b,c \in U$ and $m < n$.

By Lemma 3.3 we have

$$\varphi_n(a,b,c)r4[a,b,c] + 4[a,b,c]r\varphi_n(a,b,c) = 0,$$

for all $a,b,c,r \in U$. Note that $4[a,b,c] \in U$ and so by Lemma 2.2 it follows that $\varphi_n(a,b,c)r[a,b,c] = 0$, for all $a,b,c,r \in U$. Thus Lemma 2.3 implies that $\varphi_n(a_1,b_1,c_1)r[a_2,b_2,c_2] = 0$, for every $r,a_i,b_i,c_i \in U$, $i = 1,2$. The proof can easily be completed using Lemmas 2.1 and 2.4. ■

As an immediate consequence of Lemma 3.5 we have the following

COROLLARY 3.6. *Under the assumptions of Lemma 3.5 we have* $d_n(abc) = \sum_{i+j+k=n} d_i(a)d_j(b)d_k(c)$, *for every* $a,b,c \in R$ *(resp.* $a,b,c \in U$*) and* $n \in \mathbb{N}$.

Now we are in a position to prove Theorem 1.2.

Proof of Theorem 1.2. First assume that R is a 2-torsion-free semiprime ring and D is a JTHD of R. Since $d_0 = id_R$ we may assume, by induction, that $d_m(ab) = \sum_{i+j=m} d_i(a)d_j(b)$, for every $a,b \in R$ and $m < n$.

Take $a,b,x \in R$ and put $w = abxab$. Using Corollary 3.6 two times we easily obtain

$$d_n(w) = \sum_{\tau=n} d_i(a)d_l(b)d_s(x)d_t(a)d_k(b),$$

where $\tau = i+l+s+t+k$. On the other hand,

$$d_n(w) = d_n((ab)x(ab)) = \sum_{i+j+k=n} d_i(ab)d_j(x)d_k(ab).$$

Comparing these two expressions of $d_n(w)$ and using the inductive assumption we obtain

$$(d_n(ab) - \sum_{i+j=n} d_i(a)d_j(b))xab + abx(d_n(ab) - \sum_{i+j=n} d_i(a)d_j(b)) = 0,$$

for every $a,b,x \in R$. Applying Lemmas 2.2 and 2.3 we have

$$(d_n(ab) - \sum_{i+j=n} d_i(a)d_j(b))xcd = 0, \text{ for every } a,b,c,d,x \in R.$$

Since R is semiprime it follows that $d_n(ab) = \sum_{i+j=n} d_i(a)d_j(b)$. The proof is complete in this case.

Now, assume that R is prime of characteristic different of 2, U is admissible and D is a JTHD of U into R. Take $a,b,x \in U$ and put again $w = abxab$. Since $4w = a(4bxa)b = (2ab)x(2ab)$, where $4bxa, 2ab \in U$ we can compute $d_n(4w)$ as above and the result easily follows using similar arguments. ■

Proof of Theorem 1.3. Assume that R is a 2-torsion-free ring and D is a JHD of R (resp. of U into R).

We claim that for any $a, b \in R$ (resp. $a, b \in U$) and $n \in \mathbb{N}$ we have

$$d_n(ab + ba) = \sum_{i+j=n} (d_i(a)d_j(b) + d_i(b)d_j(a)). \tag{2}$$

In fact, by assumption $d_n(a^2) = \sum_{i+j=n} d_i(a)d_j(a)$. The claim follows from the relations

$$\begin{aligned} d_n((a+b)^2) &= \sum_{t+u=n} d_t(a+b)d_u(a+b) = \\ &= \sum_{t+u=n} (d_t(a)d_u(a) + d_t(a)d_u(b) + d_t(b)d_u(a) + d_t(b)d_u(b)) \end{aligned}$$

and

$$\begin{aligned} d_n((a+b)^2) &= d_n(a^2 + ab + ba + b^2) = d_n(a^2) + d_n(ab+ba) + d_n(b^2) = \\ &= d_n(ab+ba) + \sum_{i+j=n} d_i(a)d_j(a) + \sum_{r+s=n} d_r(b)d_s(b). \end{aligned}$$

Now put $w = (a(ab+ba) + (ab+ba)a)$. Using (2) we have

$$\begin{aligned} d_n(w) &= \sum_{i+j=n} d_i(a)d_j(ab+ba) + \sum_{i+j=n} d_i(ab+ba)d_j(a) = \\ &= \sum_{i+j=n} \sum_{r+s=j} d_i(a)d_r(a)d_s(b) + 2 \sum_{i+j+k=n} d_i(a)d_j(b)d_k(a) + \\ &\quad + \sum_{i+j=n} \sum_{k+l=i} d_k(b)d_l(a)d_j(a). \end{aligned}$$

Also,

$$d_n(w) = d_n((a^2b + ba^2) + 2aba) = d_n(a^2b + ba^2) + 2d_n(aba) =$$

$$2d_n(aba) + \sum_{i+j=n} \sum_{r+s=i} d_r(a)d_s(a)d_j(b) + \sum_{i+j=n} d_i(b) \sum_{k+l=j} d_k(a)d_l(a).$$

It follows that $2d_n(aba) = 2\sum_{i+j+k=n} d_i(a)d_j(b)d_k(a)$, and the result follows because R is 2-torsion-free. ∎

The following example shows that the result of Corollary 1.4 for Lie ideals is no more true in the semiprime case. This example is due to M. Brešar who kindly allowed us to include it here.

Example. Let F be a field and let S be the ring of all polynomials over F in non-commuting indeterminates x and y with zero constant term. Further let T be any semiprime algebra over F and put $R = S \times T$. Thus $U = S \times 0$ is an admissible Lie ideal of R.

Define $g : R \to R$ by $g(p,t) = (a-b)t_0 \in T \subset R$, where $p \in S$, $t \in T$, a (resp. b) is the coefficient at xy (resp. yx) in the polynomial p and $t_0 \neq 0$ is a fixed element in T. Then it is easy to see that $g((p,t)^2) = 0$, for every $(p,t) \in R$. However $g(xy, 0) = t_0 \neq 0$. Hence $g\,|_U$ is a Jordan derivation of U into R and is not a derivation of U into R.

Now we further assume that $t_0^2 = 0$, so that $g(u)^2 = 0$ for every $u \in U$, and define $d_0 = id$, $d_1 = g$ and $d_n = 0$, for every $n \geq 2$. Then $D = (d_i)_{i \in \mathbb{N}}$ is a JHD of U into R which is not a HD of U into R.

Remark. The result proved in ([1], Theorem) also holds for Lie ideals contained in the center of R. Actually, in this case the Theorem is trivial. We were unable to answer the question on whether the corresponding result is also true for higher derivations.

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