THE USE OF AN ARTICULATE MECHANICAL ARM TYPE ROBOT BUILT FROM MATERIALS OF LOW COST AS A SUPPORTING TOOL FOR TEACHING AT THE UNDERGRADUATE LEVEL: THE RESOLUTION OF THE DIRECT AND INVERSE KINEMATIC MODELS IN THE BIDIMESIONAL CASE

Claus Haetinger[1], Werner Haetinger[2], Eduardo Dullius[3], Deivi Marcolin, Jeferson Schmidt, Mateus Ohse[4] and Niumar Klein

Abstract *— It is a proposal of using of articulate mechanical arm type (with 4 degrees of freedom, motors of continuous current, magnetic claw) robots, built from materials of low cost, as a supporting tool for teaching-learning of Mathematics at the undergraduation level. We developed an algebraic bidimensional control system for the direct and inverse kinematic models, facilitating the programming of its paths (in substitution to the heuristic algorithms), describing the transformations of coordinates associated to the movement of each articulation of the robot. This modelling proved to be simple and, in spite of the bidimensional case, allows to the student a more dynamic and practical learning of the concepts developed in Lineal Algebra. It also shows with more details the applications of the Differential Equations; it interacts theory and practice in Programming Language, besides consisting in a materialization of a real problem that will be part of the student's professional life.*

Index Terms — Engineering Education, Linear Algebra, Mathematical Modelling, Robotics.

1. MOTIVATION

The control of articulate mechanisms, as the robots, is strongly based on mathematical operations [6] to execute the coordinated movement of its movable elements in certain paths. Therefore, it is natural to use a mechanical arm to develop the contents of Lineal algebra and Analytic Geometry with the undergraduate students. The Mathematics, the Electronics, the Mechanics and the Computer science are the bases of the robotics, that has been used, more and more, in the activities of automated control, mainly in industrial processes. In the industry, the robots are used to manipulate objects and to move them of one place for other in factory ground accurately [10]. They are also appropriate to manipulate objects in unhealthy or polluted atmospheres, for instance containing toxicant gases. The problem is that the robots have a very high cost, restricting their use in class room in most of the schools and universities.

One of the educator's challenges is to turn the teaching-learning process more significant. The world out of the class room is much more attractive for the student than the activities that he accomplishes inside of it, what has been reducing your interest for the study, impelling the teacher to seek new strategies to innovate your classes. To fill out this gap, two robots were built in the development of a research that had the students' grant holders of scientific initiation participation, besides having the voluntary students' participation joined to the project in several moments of the research. An articulate mechanical arm type robot opens possibility to explore a wide set of contents in undergraduate courses.

This work presents a proposal of using of articulate mechanical arm type robots, built from materials of low cost. The robots were totally projected and built in UNIVATES, from the mechanical and electronic parts until the software.

The article is organized in the following way: in the Section 2 it is made a description of the construction and of the robot's characteristics; in the Section 3 we approached the mathematical modelling, describing the involved algebraic operations; in the Section 4 we described an ideal path for one specific problem, and in the Section 5 we presented the conclusions and future works.

2. DESCRIPTION OF THE ROBOT

The first robot of UNIVATES was built with motors of continuous current of constant magnetic field acquired in junk-dealer trade. The robot's structure was made in wood by being a quite light material and easy to cut, to hole, to sand and to be handled by the students grant holders using the available equipments in the Institution. The axes of the robot's joints were built on bicycle cubes, since they are cheap and of easy installation. The controlling plates were also developed in UNIVATES

[1] Claus Haetinger, Research Professor, Centro Universitário UNIVATES, Departamento III, Rua Avelino Tallini, nr. 171, ZIP Code 95900-000, Lajeado, RS, Brazil, chaet@univates.br .
[2] Werner Haetinger, Research Professor, Centro Universitário UNIVATES, werner@univates.br . Partially supported by Fundação de Amparo à Pesquisa do Estado do Rio Grande do Sul (FAPERGS, Brazil).
[3] Supported by a fellowship (scientific initiation) granted by Centro Universitário UNIVATES (Brazil), edullius@univates.br .
[4] Supported by a fellowship (scientific initiation) granted by Centro Universitário UNIVATES (Brazil), ozzy_dm@univates.br .

using recycled electronic components. The robot has a fixed base, a rotative vertical support, an upper arm (L1), a forearm (L2), a rotative pulse and a magnetic claw in your extremity. Therefore, with 4 degrees of freedom (dof, for short) [5]. The revolution joints of the forearm and of the upper arm lie in the same plane and the robot's claw holds objects through electromagnetism (Figure 1).

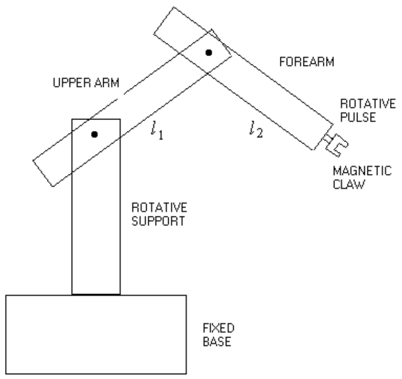

FIGURE. 1
ARTICULATE MECHANICAL ARM TYPE ROBOT.

3. The Mathematical Modelling

We studied the mathematical formalism [2], turning it applicable to our mechanical arm, so that it could be used in the undergraduate courses of Engineering and Mathematics.

We described here a bidimensional version of the movement of an articulate mechanical arm type robot [7]-[9].

Let θ_1 and θ_2 be the angles of rotation of the arm in the plane, called *control angles*, and let be l_1 and l_2 be the lengths of the segments (also called *links*, or *rigid parts*): the upper arm and the forearm, respectively (Figure 2).

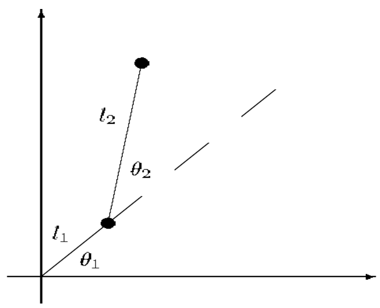

FIGURE. 2
SCHEMATIZED MODEL OF THE BIDIMENSIONAL MECHANICAL ARM.

Let $P=(x,y)$ be a point of the XOY plane that we want to take with the robot. We say that P is the *terminal link*. Now we show the parametric equations of the progress in this simplified model:

$$\begin{aligned} x &= l_1 * \cos(\theta_1) + l_2 * \cos(\theta_1 + \theta_2) \\ y &= l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \theta_2) \end{aligned} \tag{1}$$

The robot should be usable in applied problems, such as to search for and to catch a certain object in the depot of an industry, or to paint a metallic plate. In this second case, the speed (and, therefore, the time) used by the arm to execute a certain movement is fundamental, to avoid that the painting will be very fine or very thickens. Already in the first case, the speed is important to avoid that a certain product will be shake in surplus and spill (or until it explodes). Then it is necessary to model the movements of the terminal link mathematically to generate the ideal paths for the mechanical arm.

Let us consider the control angles as function of the time t. Therefore $\theta_1= \theta_1(t)$ and $\theta_2= \theta_2(t)$. So (1) can be written as:

$$\begin{aligned} x &= l_1 * \cos(\theta_1(t)) + l_2 * \cos(\theta_1(t) + \theta_2(t)) \\ y &= l_1 * sin(\theta_1(t)) + l_2 * sin(\theta_1(t) + \theta_2(t)) \end{aligned} \tag{2}$$

Suppose that the arm extends horizontally along the positive axis OX starting from $t=0$ and that the links 1 and 2 rotate to constant rates of ω_1 *rad/s* and ω_2 *rad/s*, respectively. Then the kinematics equations are given by:

$$\begin{aligned} x &= l_1 * \cos(\omega_1 * t) + l_2 * \cos((\omega_1 + \omega_2) * t) \\ y &= l_1 * sin(\omega_1 * t) + l_2 * sin((\omega_1 + \omega_2) * t) \end{aligned} , \tag{3}$$

or by:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\omega_1 * t) & \cos((\omega_1 + \omega_2) * t) \\ sin(\omega_1 * t) & sin((\omega_1 + \omega_2) * t) \end{bmatrix} * \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \tag{4}$$

Example: using low cost softwares [3], we can simulate several situations and we can verify the inclusion area and the pattern of movements of the terminal link in a certain interval of time, as we modify the values of ω_i, l_i, $i=1,2$ (Figures 3 and 4).

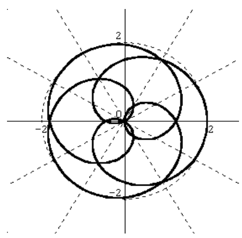

FIGURE. 3
PATTERN OF MOVEMENT OF THE CLAW WITH *l1=l2=1, w1=2 rad/s* AND *w2=3 rad/s*.

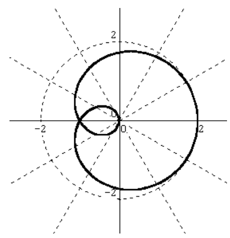

FIGURE. 4
PATTERN OF MOVEMENT OF THE CLAW WITH *l1=l2=1, w1=w2=1 rad/s*.

4. THE DIRECT AND INVERSE KINEMATIC MODELS

Our robot should paint a metallic plate, making vertical movements, from the base upward. After the painting of each strip, it should come back to the base and to move horizontally for the new strip.

For instance, let us suppose that the metallic plate has *90 cm* of width by *150 cm* of height. We should opt for a robot with links of enough length to paint the whole surface of the plate. Let us say that we have opted for a robot with two links of same length, $l_1=l_2=90$ *cm*, whose base is positioned near the left lower corner of the plate (Figure 5a). Note that the enlarged links of the arm embrace a radio of *180 cm*. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\omega_1 * t) & \cos((\omega_1+\omega_2)*t) \\ sin(\omega_1 * t) & sin((\omega_1+\omega_2)*t) \end{bmatrix} * \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$

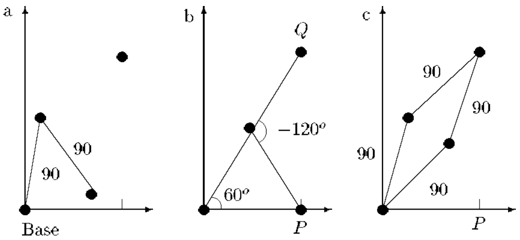

FIGURE. 5
THE ROBOT´S POSITIONINGS TO PAINT THE METALLIC PLATE.

We began painting the right hand side of the plate from a point P=(90,0) even Q=(90,150). Using some trigonometry and basic geometry, we can determine the position of the terminal link at the point P, choosing angles of control $\theta_1 = \frac{\pi}{3}$ e $\theta_2 = \frac{-2\pi}{3}$ (Figure 5b).

In a general way, the main question is how to find the control correspondents' angles to each point Q=(u,v) in the plane. For such, we used the kinematics equations:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & \cos(\theta_1+\theta_2) \\ \sin(\theta_1) & \sin(\theta_1+\theta_2) \end{bmatrix} \cdot \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}. \quad (5)$$

Therefore, to put the terminal link at the point Q=(u,v), the controlling angles should satisfy the equations

$$\begin{aligned} u &= l_1 \cdot \cos(\theta_1) + l_2 \cdot \cos(\theta_1+\theta_2) \\ v &= l_1 \cdot \sin(\theta_1) + l_2 \cdot \sin(\theta_1+\theta_2), \end{aligned} \quad (6)$$

where l_1 and l_2 are the lengths of the upper arm and of the forearm, respectively.

From the second equation of (6) we have that $l_2 * sin(\theta_1+\theta_2) = v - l_1 * sin(\theta_1)$. Squaring the left and the right hand side (LHS and RHS, for short) of this expression, and applying the fundamental relationship of the trigonometry (FRT, for short) for $(\theta_1+\theta_2)$, we obtain

$$l_2^2 - l_2^2 \cdot \cos^2(\theta_1+\theta_2) = v^2 - 2vl_1 \cdot \sin(\theta_1) + l_1^2 \cdot \sin^2(\theta_1). \quad (7)$$

From the first equation of (6), we have that $\cos(\theta_1+\theta_2) = \frac{u}{l_2} - \frac{l_1}{l_2} * \cos(\theta_1)$.

Again, squaring the LHS and the RHS of this expression, we obtain

$$\cos^2(\theta_1+\theta_2) = \left(\frac{u}{l_2}\right)^2 - \frac{2ul_1}{l_2^2} \cdot \cos(\theta_1) + \left(\frac{l_1}{l_2}\right)^2 \cdot \cos^2(\theta_1).$$

Substituting this identity in (7), it results

$$l_2^2 - u^2 + 2ul_1 \cdot \cos(\theta_1) - l_1^2 \cdot \cos^2(\theta_1)) = v^2 - 2vl_1 \cdot \sin(\theta_1) + l_1^2 \cdot \sin^2(\theta_1).$$

Using the FRT for θ_1 in the equality above, we obtain

$$2vl_1 \cdot \sin(\theta_1) = u^2 + v^2 + l_1^2 - l_2^2 - 2ul_1 \cdot \cos(\theta_1).$$

Squaring both the hand sides of the expression, and using the FRT for θ_1, we obtain the equation:

$$(4u^2l_1^2 + 4v^2l_1^2) \cdot \cos^2(\theta_1) - 4ul_1 \cdot (u^2+v^2+l_1^2-l_2^2) \cdot \cos(\theta_1) + (u^2+v^2+l_1^2-l_2^2)^2 - 4v^2l_1^2 = 0.$$

Let now

$$\begin{aligned} A &= 4u^2l_1^2 + 4v^2l_1^2, \\ B &= -4ul_1 \cdot (u^2+v^2+l_1^2-l_2^2), \\ C &= (u^2+v^2+l_1^2-l_2^2)^2 - 4v^2l_1^2 \end{aligned}$$

be.

The equation above is written as $A * \cos^2(\theta_1) + B * \cos(\theta_1) + C = 0$. Therefore we have two solutions:

$$\begin{aligned} \theta_{1_1} &= \arccos\left(\frac{-B+\sqrt{B^2-4AC}}{2A}\right) \ rad \\ \theta_{1_2} &= \arccos\left(\frac{-B-\sqrt{B^2-4AC}}{2A}\right) \ rad. \end{aligned} \quad (8)$$

It is enough now to substitute each one of these angles in the first equation of (6) and to solve them, obtaining

$$\begin{aligned} \theta_{2_1} &= -\theta_{1_1} + \arccos\left(\frac{u - l_1 \cdot \cos(\theta_{1_1})}{l_2}\right) \ rad, \\ \theta_{2_2} &= -\theta_{1_2} + \arccos\left(\frac{u - l_1 \cdot \cos(\theta_{1_2})}{l_2}\right) \ rad. \end{aligned} \quad (9)$$

It is easy to see that once defined the position of the terminal link, there will be always two ideal positions for the joints (Figure 5c).

Defined the extreme point of the forearm, *Q=(u,v)*, and determined the angles θ_{11} and θ_{21}, we can determine the extreme point of the upper arm, *P=(a,b)*, through parametric equations:

$$\begin{aligned} a &= l_1 \cdot \cos(\theta_{1_1}) \\ b &= l_1 \cdot \sin(\theta_{1_1}). \end{aligned}$$

The link of the upper arm is given by the equation of the straight line that goes by the points *O=(0,0)* and *P=(a,b),* that is:

$$r_1 : \quad y = \frac{b}{a} \cdot x, \quad x \in [0, a].$$

On the other hand, the link of the forearm is given by the equation of the straight line that goes by the points *P=(a,b)* and *Q=(u,v):*

$$r_2 : \quad y = \frac{(v-b)}{u-a} \cdot x + \frac{(ub-av)}{u-a}, \quad x \in [a, u].$$

Let us now study how to program the computer to paint. For instance, to make the painting in the vertical sense, from the point *S=(u,0)* until the point *Q=(u,v),* with constant speed, it is enough to consider the graph of r_1 in the interval *[0,a]* and the graph of r_2 in the interval *[a,u]*.

Observe that exists a limitation for the coordinates *u* and *v*, because from the lengths of the links and since we are in the bidimensional plane case.

Finally, let us study how to program the computer to paint the segment of vertical straight line among two points.

We know that, besides moving the terminal link on the vertical straight line, we needed to control your speed. In our case, the robot is moved with a speed constant v_{vert}. We needed now to determine the rates of rotation $d\theta_1/dt$ and $d\theta_2/dt$ (in *rad/sec*) so that $du/dt=0$ and $dv/dt= v_{vert}$. The first condition guarantees that the terminal link moves vertically (because there is not horizontal speed), while the second condition guarantees a movement upward to the speed of v_{vert} cm/s.

Differentiating the kinematic equations (5), we obtain:

$$\begin{aligned} \frac{du}{dt} &= -l_1 \cdot \sin(\theta_1) \cdot \frac{d\theta_1}{dt} - l_2 \cdot \sin(\theta_1 + \theta_2) \cdot \left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right) \\ \frac{dv}{dt} &= l_1 \cdot \cos(\theta_1) \cdot \frac{d\theta_1}{dt} + l_2 \cdot \cos(\theta_1 + \theta_2) \cdot \left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right) \end{aligned} \qquad (10)$$

Using the kinematic equations again, we can simplify (10) and then we can use the initial conditions $du/dt=0$ and $dv/dt= v_{vert}$ to solve them, obtaining:

$$\begin{aligned} -v \cdot \frac{d\theta_1}{dt} - l_2 \cdot \sin(\theta_1 + \theta_2) \cdot \frac{d\theta_2}{dt} &= 0 \\ u \cdot \frac{d\theta_1}{dt} + l_2 \cdot \sin(\theta_1 + \theta_2) \cdot \frac{d\theta_2}{dt} &= v_{vert}. \end{aligned} \qquad (11)$$

The equations (11) are used in the following algorithms:

1. The Kinematics Direct Model: the model consists of the description of the position and of the orientation of the claw in function of the variables of the links. To obtain this model, it associates to each link of the robot a system of coordinates. We have a unique solution for each given position.

- In each instant *t*, the robot transmits the angles of control θ_1 and θ_2 of your links for the computer;
- The computer uses the kinematic equations (5) to calculate the coordinates *u* and *v* of the terminal link;
- With these data, the θ_1, θ_2 , *u* and *v* values are substituted in (11) to produce two equations in the unknowns $d\theta_1/dt$ and $d\theta_2/dt$;
- The computer solves the equations and determines the rotation rates demanded for the links.

2. The Kinematics Inverse Model: the model consists of the determination of the variables starting from the position and the orientation of the claw. There are two solutions for each given position.

- In each instant *t*, the robot transmits the coordinates *u* and *v* of the terminal link;
- The computer uses the equations (8) and (9) to calculate the control angles coordinates θ_1 and θ_2 of your links;
- With these data, the θ_1, θ_2 , *u* and *v* values are substituted in (11) to produce two equations in the unknowns $d\theta_1/dt$ and $d\theta_2/dt$;
- The computer solves the equations and determines the rotation rates demanded for the links.

Example: with the data of our last example, we can evaluate the rotation rates when the terminal link moves with a constant velocity of *1 cm/sec*. It is easy to see that the rates will be *0.0111 rad/sec* and *0.0128 rad/sec.*

5. CONCLUSIONS AND FUTURE ACTIVITIES

The robot's use turns more tangible to the student the understanding of the concepts of Linear Algebra and of the Analytic Geometry, it shows with more clarity the applications of the Differential Equations, it allows an interaction among theory and practice in the Programming Language and Numerical Methods, besides consisting in a materialization of a real problem that will take part of the student's professional life.

As future work we are researching the models described above in the three-dimensional case and their implications of the matrix-theoretical point of view, as well as the computational implements. It is being also researched the elaboration of a program to do graphic simulation of the robot's movements, starting from the matrix calculations, which will be developed by students.

REFERENCES

[1] Anton, H.; Rorres, C., *Álgebra Linear com aplicações*, Bookman, 2001.

[2] Fu, K. S.; Gonzalez, R. C.; Lee, C. S. G.*; Robotics: control, censing, vision and intelligence*, McGraw Hill, 1987.

[3] Haetinger, C., *Grupo de estudos no uso de aplicativos matemáticos computacionais de baixo custo no ensino da graduação*, Lajeado-RS, UNIVATES, PROPEX, 2002 (unpublished).

[4] Haetinger, C.; Haetinger, W.; "Robótica em sala de aula: modelagem, descrição e geração de trajetórias ideais para o braço mecânico – o caso bidimensional", Florianópolis-SC*, I Encontro Regional de Matemática Aplicada e Computacional de Santa Catarina* (I ERMAC-SC), 2003 (to appear).

[5] Haetinger, W.; *"Estudo e fabricação de um braço mecânico controlado por computador utilizando componentes de baixo custo"*, Lajeado-RS, UNIVATES, PROPEX, 2000 (unpublished).

[6] Latombe, J.C.; *"Robot motion planning"*, Kluwer Academic Publishers, 1991.

[7] Meyer, W.; "Moving a planar robot arm", *MAA Notes*, No. 29, The Mathematical Association of America, 1993

[8] Neto, F.D.M.; Pereira, F.R.; "Modelagem na indústria: uma viagem das ciências básicas à Engenharia", Belo Horizonte-MG, *I Bienal da SBM*, Depto. De Matemática –Icex/UFMG, 2002.

[9] Pazos, F.; *"Automação de sistemas & robótica",* São Paulo, Axcel Books, 2002.

[10] Salant, M.A.; *"Introdução à robótica",* McGraw-Hill, São Paulo, 1990.